

## CHAPTER SUMMARY

## • Functions

Definition: a rule which takes certain numbers as inputs and assigns to each input exactly one output number.

Function notation,  $y = f(x)$ .

Use of vertical line test.

## • Average rate of change

Average rate of change of  $Q = f(t)$  on  $[a, b]$  is

$$\frac{\Delta Q}{\Delta t} = \frac{f(b) - f(a)}{b - a}$$

Increasing, decreasing functions; identifying from average rate of change.

## • Linear Functions

Value of  $y$  changes at constant rate.

## • Formulas for Linear Functions

Slope-intercept form:  $y = b + mx$ .

Point-slope form:  $y - y_0 = m(x - x_0)$ .

Standard form:  $Ax + By + C = 0$ .

## • Properties of Linear Functions

Interpretation of slope, vertical and horizontal intercepts.

Intersection of lines: Solution of equations.

Parallel lines:  $m_1 = m_2$ .

Perpendicular lines:  $m_1 = -\frac{1}{m_2}$ .

## • Fitting Lines to Data

Linear regression; correlation. Interpolation, extrapolation; dangers of extrapolation.

## REVIEW EXERCISES AND PROBLEMS FOR CHAPTER ONE

## Exercises

In Exercises 1–5 a relationship is given between two quantities. Are both quantities functions of the other one, or is one or neither a function of the other? Explain.

1.  $7w^2 + 5 = z^2$     2.  $y = x^4 - \frac{1}{x}$     3.  $m = \sqrt{t}$

4.

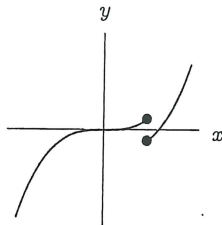


Figure 1.60

5. The number of gallons of gas,  $g$ , at \$2 per gallon and the number of pounds of coffee,  $c$ , at \$10 per pound that can be bought for a total of \$100.
6. In 2005, you have 40 CDs in your collection. In 2008, you have 120 CDs. In 2012, you have 40. What is the average rate of change in the size of your CD collection between
- (a) 2005 and 2008?    (b) 2008 and 2012?  
(c) 2005 and 2012?
7. Find the average rate of change of  $f(x) = 3x^2 + 1$  between the points
- (a)  $(1, 4)$  and  $(2, 13)$     (b)  $(j, k)$  and  $(m, n)$   
(c)  $(x, f(x))$  and  $(x+h, f(x+h))$

In Exercises 8–9, could the table represent a linear function?

8. 

$\lambda$	1	2	3	4	5
$q(\lambda)$	2	4	8	16	32

 9. 

$t$	3	6	9	12	15
$a(t)$	2	4	6	8	10

Problems 10–12 give data from a linear function. Find a formula for the function.

10.

$x$	200	230	300	320	400
$g(x)$	70	68.5	65	64	60

11.

$t$	1.2	1.3	1.4	1.5
$f(t)$	0.736	0.614	0.492	0.37

12.

$t$	5.2	5.3	5.4	5.5
$f(t)$	73.6	61.4	49.2	37

In Exercises 13–14, which line has the greater

- (a) Slope?    (b)  $y$ -intercept?

13.  $y = 7 + 2x$ ,     $y = 8 - 15x$

14.  $y = 5 - 2x$ ,     $y = 7 - 3x$

Are the lines in Exercises 15–18 perpendicular? Parallel? Neither?

15.  $y = 5x + 2$     $y = 2x + 5$

16.  $y = 14x - 2$     $y = -\frac{1}{14}x + 2$

17.  $y = 3x + 3$     $y = -\frac{1}{3}x + 3$

18.  $7y = 8 + 21x$ ;  $9y = 77 - 3x$

### Problems

Find formulas for the linear functions in Problems 19–22.

19. The graph of  $f$  contains  $(-3, -8)$  and  $(5, -20)$ .

20.  $g(100) = 2000$  and  $g(400) = 3800$

21.  $P = h(t)$  gives the size of a population that begins with 12,000 members and grows by 225 members each year.

22. The graph of  $h$  intersects the graph of  $y = x^2$  at  $x = -2$  and  $x = 3$ .

23. Find the equation of the line parallel to  $3x + 5y = 6$  and passing through the point  $(0, 6)$ .

24. Find the equation of the line passing through the point  $(2, 1)$  and perpendicular to the line  $y = 5x - 3$ .

25. Find the equations of the lines parallel to and perpendicular to the line  $y + 4x = 7$ , and through the point  $(1, 5)$ .

26. You have zero dollars now and the average rate of change in your net worth is \$5000 per year. How much money will you have in forty years?

27. A flight costs \$10,000 to operate, regardless of the number of passengers. Each ticket costs \$127. Express profit,  $\pi$ , as a linear function of the number of passengers,  $n$ , on the flight.

28. Table 1.42 gives the ranking  $r$  for three different names—Hannah, Alexis, and Madison. Of the three names, which was most popular and which was least popular in

(a) 1995?

(b) 2004?

**Table 1.42** Ranking of names—Hannah ( $r_h$ ), Alexis ( $r_a$ ), and Madison ( $r_m$ )—for girls born between 1995 ( $t = 0$ ) and 2004 ( $t = 9$ )<sup>25</sup>

$t$	0	1	2	3	4	5	6	7	8	9
$r_h$	7	7	5	2	2	2	3	3	4	5
$r_a$	14	8	8	6	3	6	5	5	7	11
$r_m$	29	15	10	9	7	3	2	2	3	3

29. Table 1.42 gives information about the popularity of the names Hannah, Madison, and Alexis. Describe in words what your answers to parts (a)–(c) tell you about these names.

(a) Evaluate  $r_m(0) - r_h(0)$ .

(b) Evaluate  $r_m(9) - r_h(9)$ .

(c) Solve  $r_m(t) < r_a(t)$ .

30. Figure 1.61 gives the depth of the water at Montauk Point, New York, for a day in November.

(a) How many high tides took place on this day?

(b) How many low tides took place on this day?

(c) How much time elapsed in between high tides?

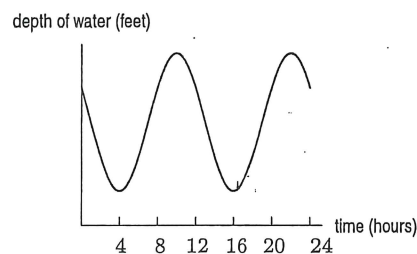


Figure 1.61

31. (a) Is the area,  $A$ , of a square a function of the length of one of its sides,  $s$ ?

(b) Is the area,  $A$ , of a rectangle a function of the length of one of its sides,  $s$ ?

32. A person's blood sugar level at a particular time of the day is partially determined by the time of the most recent meal. After a meal, blood sugar level increases rapidly, then slowly comes back down to a normal level. Sketch a person's blood sugar level as a function of time over the course of a day. Label the axes to indicate normal blood sugar level and the time of each meal.

<sup>25</sup>Data from the SSA website at [www.ssa.gov](http://www.ssa.gov), accessed January 12, 2006.



33. Many people think that hair growth is stimulated by haircuts. In fact, there is no difference in the rate hair grows after a haircut, but there *is* a difference in the rate at which hair's ends break off. A haircut eliminates dead and split ends, thereby slowing the rate at which hair breaks. However, even with regular haircuts, hair will not grow to an indefinite length. The average life cycle of human scalp hair is 3-5 years, after which the hair is shed.<sup>26</sup>

Judy trims her hair once a year, when its growth is slowed by split ends. She cuts off just enough to eliminate dead and split ends, and then lets it grow another year. After 5 years, she realizes her hair won't grow any longer. Graph the length of her hair as a function of time. Indicate when she receives her haircuts.

34. At the end of a semester, students' math grades are listed in a table which gives each student's ID number in the left column and the student's grade in the right column. Let  $N$  represent the ID number and the  $G$  represent the grade. Which quantity,  $N$  or  $G$ , must necessarily be a function of the other?

35. A price increases 5% due to inflation and is then reduced 10% for a sale. Express the final price as a function of the original price,  $P$ .

36. An 8-foot tall cylindrical water tank has a base of diameter 6 feet.

- How much water can the tank hold?
- How much water is in the tank if the water is 5 feet deep?
- Write a formula for the volume of water as a function of its depth in the tank.

37. Figure 1.62 shows the fuel consumption (in miles per gallon, mpg) of a car traveling at various speeds.

- How much gas is used on a 300 mile trip at 40 mph?
- How much gas is saved by traveling 60 mph instead of 70 mph on a 200 mile trip?
- According to this graph, what is the most fuel-efficient speed to travel? Explain.

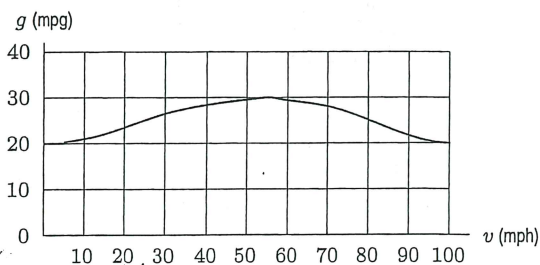


Figure 1.62

<sup>26</sup> *Britannica Micropaedia* vol. 5. (Chicago: Encyclopaedia Britannica, Inc., 1989).

38. Academics have suggested that loss of worker productivity can result from sleep deprivation. An article in the Sunday, September 26, 1993, *New York Times* quotes David Poltrack, the senior vice president for planning and research at CBS, as saying that seven million Americans are staying up an hour later than usual to watch talk show host David Letterman. The article goes on to quote Timothy Monk, a professor at the University of Pittsburgh School of Medicine, as saying "... my hunch is that the effect [on productivity due to sleep deprivation among this group] would be in the area of a 10 percent decrement." The article next quotes Robert Solow, a Nobel prize-winning professor of economics at MIT, who suggests the following procedure to estimate the impact that this loss in productivity will have on the US economy—an impact he dubbed "the Letterman loss." First, Solow says, we find the percentage of the work force who watch the program. Next, we determine this group's contribution to the gross domestic product (GDP). Then we reduce the group's contribution by 10% to account for the loss in productivity due to sleep deprivation. The amount of this reduction is "the Letterman loss."

- The article estimated that the GDP is \$6.325 trillion, and that 7 million Americans watch the show. Assume that the nation's work force is 118 million people and that 75% of David Letterman's audience belongs to this group. What percentage of the work force is in Dave's audience?
- What percent of the GDP would be expected to come from David Letterman's audience? How much money would they have contributed if they had not watched the show?
- How big is "the Letterman Loss"?

39. There are  $x$  male job-applicants at a certain company and  $y$  female applicants. Suppose that 15% of the men are accepted and 18% of the women are accepted. Write an expression in terms of  $x$  and  $y$  representing each of the following quantities:

- The total number of applicants to the company.
- The total number of applicants accepted.
- The percentage of all applicants accepted.

40. You start 60 miles east of Pittsburgh and drive east at a constant speed of 50 miles per hour. (Assume that the road is straight and permits you to do this.) Find a formula for  $d$ , your distance from Pittsburgh as a function of  $t$ , the number of hours of travel.

Table 1.43 gives the cost,  $C(n)$ , of producing a certain good as a linear function of  $n$ , the number of units produced. Use the table to answer Problems 41–43.

Table 1.43

$n$ (units)	100	125	150	175
$C(n)$ (dollars)	11000	11125	11250	11375

41. Evaluate the following expressions. Give economic interpretations for each.

(a)  $C(175)$                       (b)  $C(175) - C(150)$   
 (c)  $\frac{C(175) - C(150)}{175 - 150}$

42. Estimate  $C(0)$ . What is the economic significance of this value?  
 43. The *fixed cost* of production is the cost incurred before any goods are produced. The *unit cost* is the cost of producing an additional unit. Find a formula for  $C(n)$  in

terms of  $n$ , given that

$$\text{Total cost} = \text{Fixed cost} + \text{Unit cost} \cdot \text{Number of units}$$

44. Sketch a family of functions  $y = -2 - ax$  for five different values of  $x$  with  $a < 0$ .  
 45. Assume  $A, B, C$  are constants with  $A \neq 0, B \neq 0$ . Consider the equation

$$Ax + By = C.$$

- (a) Show that  $y = f(x)$  is linear. State the slope and the  $x$ - and  $y$ -intercepts of  $f(x)$ .  
 (b) Graph  $y = f(x)$ , labeling the  $x$ - and  $y$ -intercepts in terms of  $A, B$ , and  $C$ , assuming  
 (i)  $A > 0, B > 0, C > 0$   
 (ii)  $A > 0, B > 0, C < 0$   
 (iii)  $A > 0, B < 0, C > 0$

## CHECK YOUR UNDERSTANDING

Are the statements in Problems 1–54 true or false? Give an explanation for your answer.

- $Q = f(t)$  means  $Q$  is equal to  $f$  times  $t$ .
- A function must be defined by a formula.
- If  $P = f(x)$  then  $P$  is called the dependent variable.
- Independent variables are always denoted by the letter  $x$  or  $t$ .
- It is possible for two quantities to be related and yet neither be a function of the other.
- A function is a rule that takes certain values as inputs and assigns to each input value exactly one output value.
- It is possible for a table of values to represent a function.
- If  $Q$  is a function of  $P$ , then  $P$  is a function of  $Q$ .
- The graph of a circle is not the graph of a function.
- If  $n = f(A)$  is the number of angels that can dance on the head of a pin whose area is  $A$  square millimeters, then  $f(10) = 100$  tells us that 10 angels can dance on the head of a pin whose area is 100 square millimeters.
- Average speed can be computed by dividing the distance traveled by the time elapsed.
- The average rate of change of a function  $Q$  with respect to  $t$  over an interval can be symbolically represented as  $\frac{\Delta Q}{\Delta t}$ .
- If  $y = f(x)$  and as  $x$  increases,  $y$  increases, then  $f$  is an increasing function.
- If  $f$  is a decreasing function, then the average rate of change of  $f$  on any interval is negative.
- The average rate of change of a function over an interval is the slope of a line connecting two points of the graph of the function.
- The average rate of change of  $y = 3x - 4$  between  $x = 2$  and  $x = 6$  is 7.
- The average rate of change of  $f(x) = 10 - x^2$  between  $x = 1$  and  $x = 2$  is the ratio  $\frac{10 - 2^2 - 10 - 1^2}{2 - 1}$ .
- If  $y = x^2$  then the slope of the line connecting the point  $(2, 4)$  to the point  $(3, 9)$  is the same as the slope of the line connecting the point  $(-2, 4)$  to the point  $(-3, 9)$ .
- A linear function can have different rates of change over different intervals.
- The graph of a linear function is a straight line.
- If a line has the equation  $3x + 2y = 7$ , then the slope of the line is 3.
- A table of values represents a linear function if  $\frac{\text{Change in output}}{\text{Change in input}} = \text{constant}$ .
- If a linear function is decreasing, then its slope is negative.